Soil temperature – analytical solutions

One dimensional physical model proposed by Van Wijk and de Vries (1963):

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|  | (1) |

where is the soil temperature (oC), is time (s), is the thermal diffusivity (m2/s), and is the soil depth (m, positive downwards). To solve this model, two boundary conditions are defined:

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|  | (2) |
|  | (3) |

where is the mean temperature (oC), is the temperature amplitude (oC), is the radial frequency for temperature variations (rad/s), and is a phase constant, or lag (rad). Using these, equation 1 can be resolved, giving:

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|  | (4) |

where is the dumping depth (m), given by:

|  |  |
| --- | --- |
|  | (5) |

This function can be used to describe variations in soil temperature at different temporal scales (e.g. daily or annual). To do this one simply supply to equations 4 and 5 the appropriate values for , , , and . Combining versions of equation with appropriate parameters enables the simultaneous simulation of temperature at daily and annual scales:

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|  | (6) |

where the subscript denotes parameters for the annual time scale and for daily.

The use and are not intuitive as they have units of radians. As generally we deal with variations over time, using the period (, s) might make easier to interpret the functions:

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|  | (7) |

The phase constant can also be converted to a time lag parameter (, s), but to make it even easier to follow its complement can be used ():

|  |  |
| --- | --- |
|  | (8) |

Using these functions, equation 6 can be written as:

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|  | (9) |

where and are the duration of one year and one day (in seconds). is the time of the day when mean daily temperature is reached since midnight (this is usually taken as mid/early morning, which puts the minimum in the small hours of the night and maximum late in the afternoon. This is unavoidable as in reality daily fluctuation in temperature are not symmetric and evenly distributed over the day, so they fit only marginally to a sinusoidal function…). Likewise is the time since the beginning of the year in which the mean annual temperature is reached, after winter (late in the year in the southern hemisphere).

Using the concept, equation 5 can be re written:

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|  | (10) |

Elias et al (2004) showed that the daily temperature amplitude is actually not constant, it varies cyclically over the year in a manner that can be described by a sinusoidal function:

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|  | (11) |

where is the long-term average of daily amplitude (oC), is the variation over time in daily amplitude, and the remaining parameters, with subscript correspond to frequency, period, and lag similar to those referred earlier. Using this definition of and solving equation 1 again, the variation in temperature over time and depth become:

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|  | (12) |

with .and , and likewise and are found using equation 5 using and , respectively. Using period instead of frequency (based on equations 7 and 8) soil temperature can be described using:

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|  | (13) |

The dumping depths as function of period are:

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|  | (14) |
|  | (15) |